IIT JEE 2020

Complex Number

Revision जरूर करे

BY OM SIR

Complex Number

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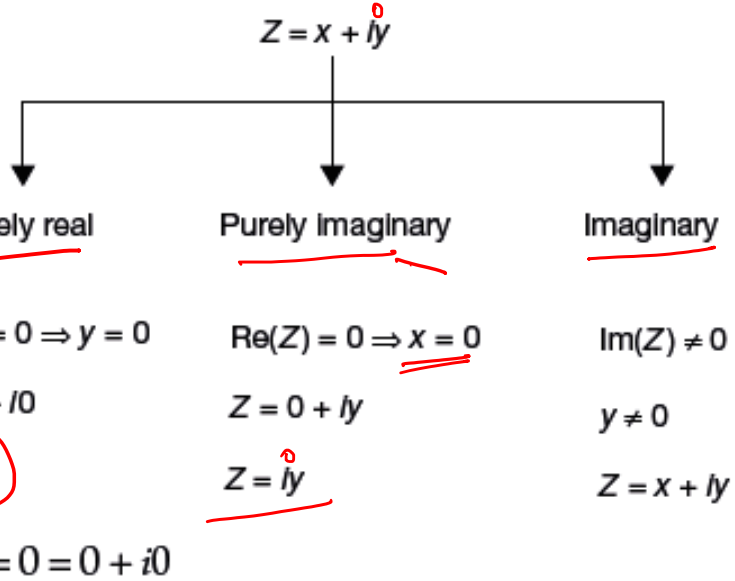
Complex number $Z = x + iy$, where $x, y \in R$

Real part of $Z = \text{Re}(Z) = x$ (Term without i)

Imaginary part of $Z = \text{Im}(z)$

$= y$ (coefficient of term present with i)

$\neq yi$



$Z = 0 = 0 + i0$

Zero is purely real, purely imaginary, real but not imaginary. Zero is also rational, integer, whole number, even number.

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Express in term of $x + iy$

(a) $\frac{1+2i}{(2+i)^2}$ (b) $\frac{1}{i}$ (c) $\frac{(1+i)(1+i^{-7})}{2}$

$\frac{1}{i^n} = -i^n$ *

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^{4n} = 1$

$i^{4n+1} = i$

$i^{4n+2} = i^2 = -1$

$i^{4n+3} = i^3 = -i$

$n \in N$

a) $x+iy = \frac{1+2i}{[(2+i)(2-i)]^2}(2-i)^2 = \frac{(1+2i)(4-1-4i)}{5^2} = \frac{(1+2i)(3-4i)}{25}$

$= \frac{3-8i^2+6i-4i}{25} = \frac{11+2i}{25}$

b) $x+iy = \frac{1}{i^9} = -i = 0-i$

c) $z = \frac{(1+i)\left(1+\frac{1}{i^{97}}\right)}{2} = \frac{(1+i)(-i+1)}{2i^{97}} = \frac{1-i^2}{-2i} = \frac{2}{-2i}$

$= \frac{-1}{i} = i$

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Equality of Complex Number

$$z_1 = z_2$$

$$\Rightarrow x_1 + iy_1 = x_2 + iy_2$$

$$x_1 = x_2 \text{ and } y_1 = y_2$$

$3+4i > 1-i$ useless

$2i > i$

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eg #

If $x, y \in R$ then find (x, y) for
$(2+3i)x^2 - (3-2i)y = 2x - 3y + 5i$

$2x^2 - 3y + i(3x^2 + 2y) = 2x - 3y + 5i$

$2x^2 - 3y = 2x - 3y$ (+) $3x^2 + 2y = 5$

$x(x-1) = 0$

$x = 0 ; x = 1$

$x = 0$ $y = 5/2$

$x = 1$ $y = 1$

$Z = 0 + \frac{5}{2}i$

$Z = 1 + i$

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If $z^2 + 2(1+2i)z - (11+2i) = 0$ find z.

$Ax^2 + Bx + C = 0$

$x = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$

$\sqrt{36} = \pm 6$

$$z = \frac{-2(1+2i) \pm \sqrt{4(1+2i)^2 + 4(11+2i)}}{2}$$

$$z = -1-2i \pm \sqrt{1-4+4i+11+2i} = -1-2i \pm \sqrt{8+6i}$$

$\sqrt{8+6i} = x+iy \quad (x, y \in R)$

$8+6i = x^2 - y^2 + 2xyi$

$x^2 - y^2 = 8$ & $2xy = 6$

$\Rightarrow y = 3/x$

$x^2 - \frac{9}{x^2} = 8$

$x^4 - 8x^2 - 9 = 0$

$(x^2-9)(x^2+1) = 0 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3; y = \pm 1$

$x^2 = -1$

$x = \pm i$

$\sqrt{8+6i} = \pm(3+i)$

$z = -1-2i \pm (3+i)$

(+) $z = 2-i$

(−) $z = -4-3i$

$z = 2-i, -4-3i$

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Geometrical Representation of complex number

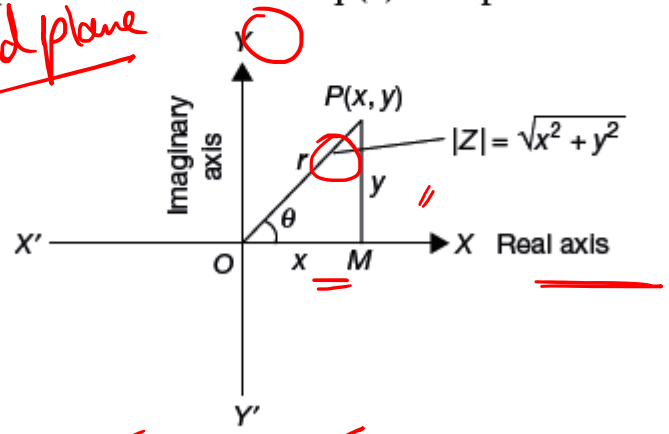
GEOMETRICAL REPRESENTATION OF COMPLEX NUMBER (Z)

The complex number $z = x + iy = (x, y)$ is represented by a point P whose coordinates are referred to rectangular axes XOX' and YOY' which are called real and imaginary axis, respectively. This plane is called Argand plane or Argand diagram or complex plane or Gaussian plane.

Let $z = x + iy$, when $x, y \in R$

OP = Modulus of $z = |z| = r = \sqrt{x^2 + y^2}$

Argument of z = Arg$(z) = \theta$ = angle made by ray OP with positive real axis = amp(z) = amplitude of z.



Where $x = r\cos\theta$ and $y = r\sin\theta$

$z = x + iy = r(\cos\theta + i\sin\theta)$

$z = x + iy$	(Cartesian Form)
$z = r(\cos\theta + i\sin\theta)$ $z = rCiS(\theta)$	(Polar From)
$z = re^{i\theta}$	(Euler Form)

$e^{i\theta} = 1 + i\theta + \frac{(i\theta)^2}{2!} + \cdots$

$= \cos\theta + i\sin\theta$

COMPUTATION OF MODULUS ($r = |z|$)

Let $z = x + iy$, when $x, y \in R$

Modulus of $z = |z| = r = \sqrt{x^2 + y^2} \neq \sqrt{x^2 + i^2y^2}$

$= \sqrt{(\text{Re}(z))^2 + (\text{Im}(z))^2}$

= Distance of complex number (z) from origin

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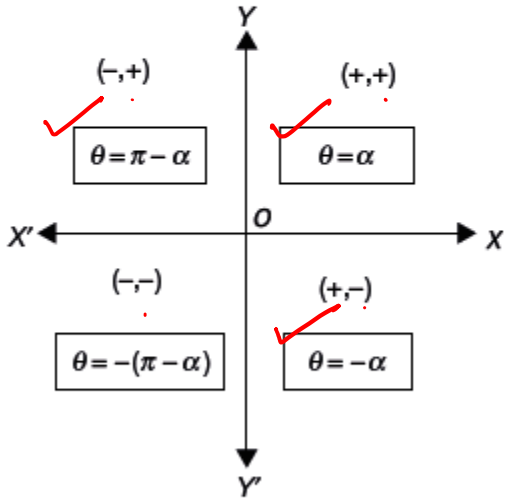
COMPUTATION OF ARGUMENT OF z (θ = ARG(z))

Let $z = x+iy$, where $x, y \in R$.

STEP-1: First, get $\alpha = \tan^{-1}\left|\frac{y}{x}\right|$ (acute angle).

STEP-2: Now locate the point (x, y) on Argand plane and calculate Arg(z).



- Amp(z) = Amplitude of z
  = Principal value of Arg(z) $\in (-\pi, \pi)$.
- Arg(z) is multivalued function.

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Compute |z| and arg(z) & convert into Euler form.

(i) $z = 1+\sqrt{3}i$ (ii) $z = -1+\sqrt{3}i$

(iii) $z = -1-\sqrt{3}i$ (iv) $z = 1-\sqrt{3}i$

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(ii) $z = -1+\sqrt{3}i$ $(-1, \sqrt{3})$ IInd

$r = \sqrt{1+3} = 2$

$\theta = \pi - \alpha = 2\pi/3$

$z = -1+\sqrt{3}i = 2e^{i2\pi/3} = 2\,cis(2\pi/3)$

$z = 1+\sqrt{3}i = x+iy$

$r = \sqrt{1+3} = 2$

$(1, \sqrt{3})$ $(+,+)$ Ist

$\alpha = \tan^{-1}\left|\frac{y}{x}\right| = \tan^{-1}\left|\frac{\sqrt{3}}{1}\right| = \pi/3$

$\theta = \alpha = \pi/3$

$z = 1+\sqrt{3}i = 2e^{i\pi/3} = 2[\cos\pi/3 + i\sin\pi/3]$

(iv) $z = 1-\sqrt{3}i$

$= 2e^{-i\pi/3}$

(iii) $z = -1-\sqrt{3}i$ $\theta = -(\pi-\alpha) = -2\pi/3$

$(-1, -\sqrt{3})$ IIIrd

$z = -1-\sqrt{3}i = 2e^{-i2\pi/3} = 2[\cos(-2\pi/3) + i\sin(-\frac{2\pi}{3})]$

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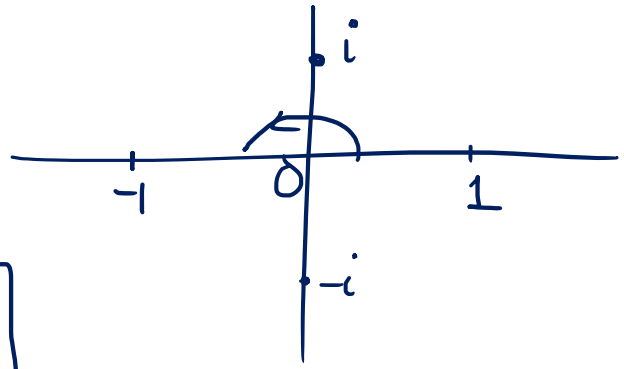
Convert into Euler form.

(i) $z = -2(\cos 30° + i \sin 30°)$

(ii) $z = 6\,(\cos 310° - i \sin 310°)$



$1 = e^{i0°}$

$i = e^{i\pi/2}$

$-1 = e^{i\pi}$

$-i = e^{-i\pi/2}$

$z = 2(e^{i\pi})\,e^{i\pi/6}$

$= 2\,e^{i(7\pi/6)}$

$z = 6\,(\cos(-310°) + i\sin(-310°))$

$= 6\,e^{-i310°} = 6\,e^{i50°}$

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Write complex number $\dfrac{\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)\left(\sqrt{3}+i\right)}{i-1}$ in rcisθ form

$$= \frac{e^{-i\pi/3}\cdot 2e^{i\pi/6}}{\sqrt{2}\,e^{i3\pi/4}}$$

$\alpha = \tan^{-1}\frac{1}{\sqrt{3}} = \pi/6$

$$= \sqrt{2}\,e^{i\left(-\pi/3+\pi/6-3\pi/4\right)}$$

$$= \sqrt{2}\,\mathrm{cis}(\ )$$

$i-1 = -1+i$

$(-1,1)$ IInd

$\alpha = \tan^{-1}\left|\frac{1}{1}\right| = \pi/4$ $\quad \theta = \pi-\alpha = 3\pi/4$

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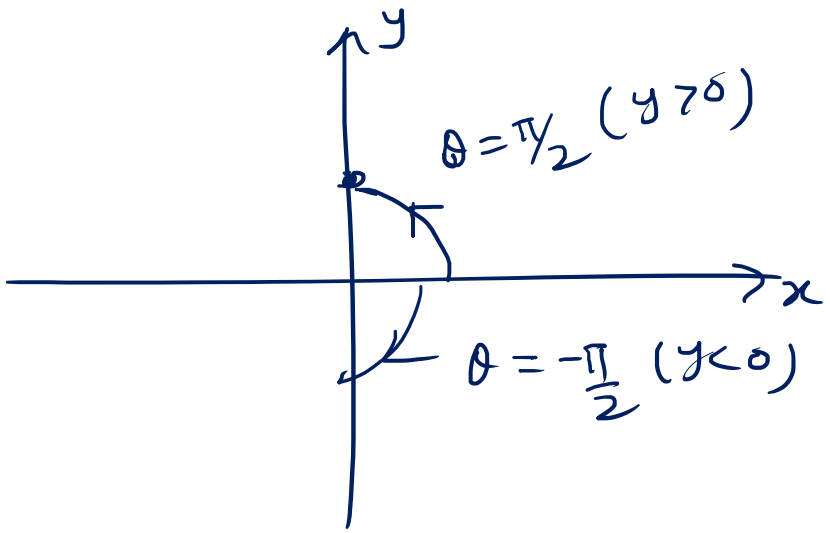
If Re(z) = 0 find arg(z)

$z = x + iy$

$x = 0 \quad z = iy$



$\arg(z) = \pm \pi/2$

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$(-\pi, \pi]$

If arg (z) = 100° find principal value of arg (z^{50})

$z = re^{i\theta}$

$z^{50} = r^{50} e^{i50\theta}$

$\arg(z^{50}) = 50\theta = 5000^\circ \notin (-180^\circ, 180^\circ]$

$= 5000^\circ - 360^\circ \times 14$

$= -40^\circ \in (-180^\circ, 180^\circ]$

$\Rightarrow$ principal value

$360^\circ \times 14$

$= 5040^\circ$

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$$\left(\frac{1+i\sin\frac{\pi}{8}+\cos\frac{\pi}{8}}{1-i\sin\frac{\pi}{8}+\cos\frac{\pi}{8}}\right)^8 \text{ equals } = \left(\frac{1+2i\sin\frac{\pi}{16}\cos\frac{\pi}{16}+2\cos^2\frac{\pi}{16}-1}{1-2i\sin\frac{\pi}{16}\cos\frac{\pi}{16}+2\cos^2\frac{\pi}{16}-1}\right)^8 = \left(\frac{2\cos\frac{\pi}{16}\left(\cos\frac{\pi}{16}+i\sin\frac{\pi}{16}\right)}{2\cos\frac{\pi}{16}\left(\cos\frac{\pi}{16}-i\sin\frac{\pi}{16}\right)}\right)^8$$

$$= \left(\frac{e^{i\pi/16}}{e^{-i\pi/16}}\right)^8 = \left(e^{i\pi/8}\right)^8 = e^{i\pi}$$

$$= \cos\pi + i\sin\pi$$

$$= -1 + 0 = -1$$

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If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$ find Im(z)

$r = \sqrt{\frac{3}{4} + \frac{1}{4}} = 1$ ↳ $\sqrt{\frac{3}{4} + \frac{1}{4}} = 1$

$\alpha = \tan^{-1}\left|\frac{1/2}{\sqrt{3}/2}\right| = \tan^{-1}\frac{1}{\sqrt{3}} = \pi/6$

$z = \left(1e^{i\pi/6}\right)^5 + \left(e^{-i\pi/6}\right)^5 = e^{i5\pi/6} + e^{-i5\pi/6}$

$= \left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) + \left(\cos\left(\frac{-5\pi}{6}\right) + i\sin\left(\frac{-5\pi}{6}\right)\right)$

$= 2\cos\frac{5\pi}{6} = 2\left(-\sqrt{3}/2\right) = -\sqrt{3}$

$Im(z) = 0$

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THEORY OF EQUATION

$$a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots\ldots + a_1 x + a_0 = 0$$

- n degree polynomial equation has exactly n roots. If n degree polynomial equation is satisfied by more than n distinct values, then it becomes an identity. Hence, it is true for all values which implies that coefficient of all variables are zero.
- If all the coefficients in equation are real, then imaginary roots are present with their conjugate.
- If not all the coefficients are real, then it is not necessary that imaginary roots are present with their conjugates.

Let the roots are $x_1, x_2, x_3, \ldots\ldots x_n$

$$a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots\ldots + a_1 x + a_0 = 0$$

$$\text{Sum of roots} = \sum x_i = -\frac{a_{n-1}}{a_n}$$

$$\sum x_i x_2 = \frac{a_{n-2}}{a_n}\,;\ \sum x_i x_2 x_3 = -\frac{a_{n-3}}{a_n}$$

$$\text{Product of roots} = x_1 x_2 x_3 \ldots\ldots x_n = (-1)^n \frac{a_0}{a_n}$$

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Eg

Let ‘z’ be complex number & a, b, c, ∈ R, if
$az^2 + bz + c + i = 0$ has purely imaginary roots, then show that $a = b^2 c$.

$z = x + iy$

0

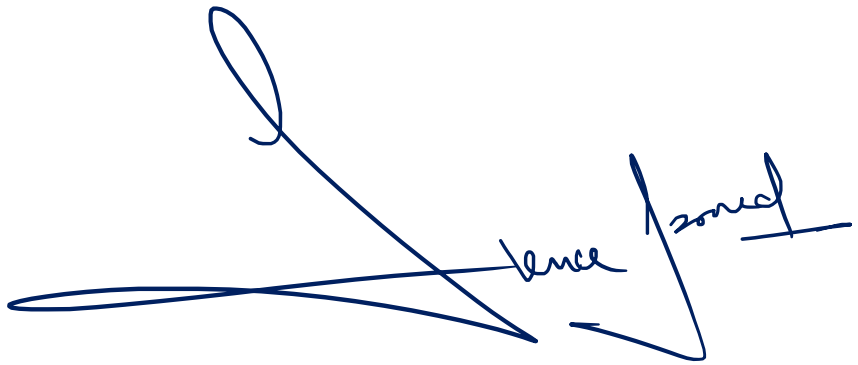
$a(iy)^2 + b(iy) + c + i = 0$

$\Rightarrow \underbrace{-ay^2 + c}_{0} + i\underbrace{(1 + by)}_{0} = 0$

$by = -1$

$c = ay^2 = a(-1/b)^2 \Rightarrow \frac{a}{b^2} = c$

$a = b^2 c$



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Let z is complex satisfying the equation $z^2 - (3 + i)z + m + 2i = 0$. Where $m \in R$. Suppose the equation has one real roots. Then find additive inverse of non-real root,

$z = x + iy$

$x^2 - (3+i)x + m + 2i = 0$

$(x^2 - 3x + m) + i(2 - x) = 0 + i0$

$x = 2$

$4 - 6 + m = 0$

$m = 2$

$\alpha + \beta = 3 + i$

$2 + \beta = 3 + i$

$\beta = 1 + i$

additive inverse

$-1 - i$

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Prove that for $\forall\, n \in N$

$$\frac{(1+\cos\theta+i\sin\theta)^n}{(1+\cos\theta-i\sin\theta)^n} = [\cos n\theta + i\sin n\theta]$$

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If $n \in N$, prove that,

$$(1 + i)^n + (1 - i)^n = 2^{n/2+1} \cos\left(\frac{n\pi}{4}\right)$$

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Conjugate Complex Number

CONJUGATE OF A COMPLEX NUMBER

If there exists a complex number $z = x + iy$, $(x, y) \in R$, then its conjugate is defined as

$\bar{z} = x - iy$ (replace i by $-i$)

$\overline{(\bar{z})} = z$

Hence, we have $\text{Re}(z) = \frac{z+\bar{z}}{2}$ and $\text{Im}(z) = \frac{z-\bar{z}}{2i}$.

Geometrically, the conjugate of z is the reflection or point image of z in the real axis.

$$W = \frac{z-2+3i}{2i-\bar{z}} \quad ; \quad \bar{\omega} = \left(\frac{z-2+3i}{2i-\bar{z}}\right) = \frac{\overline{(z-2+3i)}}{\overline{(2i-\bar{z})}}$$

$$= \frac{\bar{z}-2-3i}{-2i-z}$$

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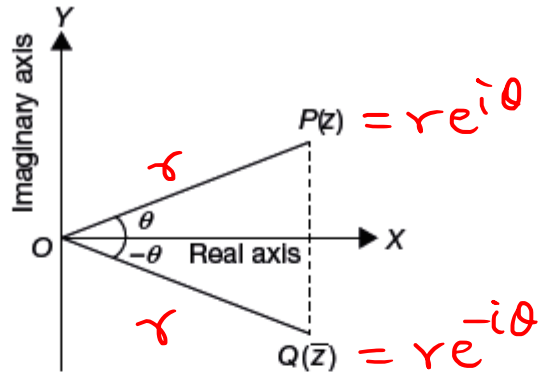

$= re^{i\theta}$

$= re^{-i\theta}$

Properties of Conjugate

$|z| = |\bar{z}|$

If z, z_1 and z_2 are existing complex numbers, then we have the following results.

$\arg(\bar{z}) = -\arg(z)$

(i) $\overline{(\bar{z})} = z$

(ii) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

(iii) $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$

(iv) $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$,

In general $\overline{z_1 . z_2 . z_3 ..... z_n} = \bar{z}_1 . \bar{z}_2 . \bar{z}_3 ..... \bar{z}_n$

(v) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$, $z_2 \neq 0$

(vi) $(\bar{z})^n = \overline{(z^n)}$

(vii) $z\bar{z} = |z|^2 = |\bar{z}|^2$ important

very powerful.

(viii) Let $z = x + iy$, $\bar{z} = x - iy$

Real part of $z = \text{Re}(z) = x = \frac{z + \bar{z}}{2}$

Imaginary part of $z = \text{Im}(z) = y = \frac{z - \bar{z}}{2i}$

(ix) $z_1\bar{z}_2 + \bar{z}_1 z_2 = 2\,\text{Re}(z_1\bar{z}_2) = 2\,\text{Re}(\bar{z}_1 z_2)$

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$z_1 = 9y^2 - 4 - 10xi$, $z_2 = 8y^2 - 20i$ z_1 & z_2 are conjugate of each other find real x & y

$z_1 = \bar{z}_2$

$9y^2 - 4 - 10xi = 8y^2 + 20i$

$9y^2 - 4 = 8y^2$ & $-10x = 20$

$x = -2$

$y^2 = 4$

$y = \pm 2$

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Properties of |z| & arg(z)

\# $z \cdot \bar{z} = |z|^2$

* $|z_1 z_2 z_3| = |z_1||z_2||z_3|$

* $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

$z_1 = r_1 e^{i\theta_1}$

$z_2 = r_2 e^{i\theta_2}$

$z_1 z_2 = r_1 r_2 e^{i(\theta_1+\theta_2)}$

$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1-\theta_2)}$

* $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$

* $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

* $\arg(z)$ — similar to log

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eg

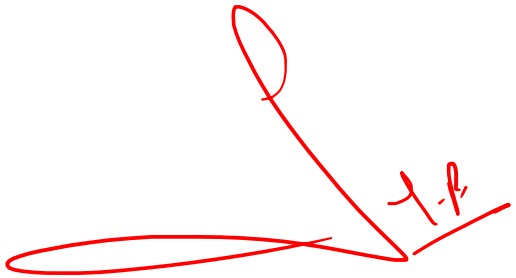
If $\frac{(a+i)^2}{2a-i} = p + qi$;

Prove that $p^2 + q^2 = \frac{(a^2+1)^2}{(4a^2+1)}$ where a, p, q $\in$ R.

$\left|\frac{(a+i)^2}{(2a-i)}\right| = |p+iq|$

$\frac{(a^2+1)}{\sqrt{4a^2+1}}$ $= \sqrt{p^2+q^2}$

square.



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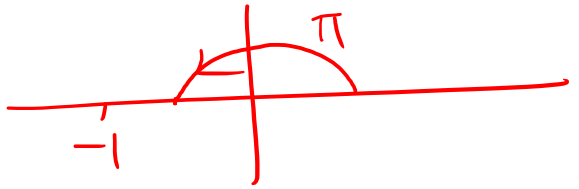
If arg(z) < 0, then arg(–z) – arg(z) equals



$$= \arg\left(\frac{-z}{z}\right) = \arg(-1)$$

$$= \pi$$

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If $\frac{7z_2}{5z_1}$ is purely imaginary find $\left|\frac{2z_1+3z_2}{2z_1-3z_2}\right|$.

$= \left|\frac{z_1(2+3z_2/z_1)}{z_1(2-3z_2/z_1)}\right|$

$\frac{7z_2}{5z_1} = 0 + iy$

$\Rightarrow \frac{z_2}{z_1} = \frac{iy}{7}$

$= \left|\frac{2+3(iy/7)}{2-3(iy/7)}\right| = 1$

$|z| = |\bar{z}|$

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If $|z_1| = 1, |z_2| = 2, |z_3| = 3$ & $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$

Find the value of $|z_1 + z_2 + z_3|$

$|z_1|^2 = 1 \Rightarrow z_1\bar{z}_1 = 1$

$|z_2|^2 = 4 \Rightarrow z_2\bar{z}_2 = 4$

$|z_3|^2 = 9 \Rightarrow z_3\bar{z}_3 = 9$

$|z_1z_2z_3\bar{z}_3 + z_1z_2\bar{z}_2z_3 + z_1\bar{z}_1z_2z_3| = 12$

$|z_1z_2z_3|\,|\bar{z}_3 + \bar{z}_2 + \bar{z}_1| = 12$

$(1)(2)(3)\;\;|\overline{z_1 + z_2 + z_3}| = 12$

$|z_1 + z_2 + z_3| = 2$

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If z_1 & z_2 are two complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular. Find $|z_1|$ if $|z_2| \neq 1$

$(\because z\bar{z} = |z|^2)$

$$\left|\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right| = 1 \Rightarrow |z_1 - 2z_2|^2 = |2 - z_1\bar{z}_2|^2$$

$$(z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) = (2 - z_1\bar{z}_2)(2 - \bar{z}_1 z_2)$$

$$\Rightarrow z_1\bar{z}_1 - 2z_1\bar{z}_2 - 2z_2\bar{z}_1 + 4z_2\bar{z}_2 = 4 - 2z_1\bar{z}_2 - 2\bar{z}_1 z_2 + z_1\bar{z}_1 z_2\bar{z}_2$$

$$\Rightarrow |z_1|^2 + 4|z_2|^2 = 4 + |z_1|^2|z_2|^2$$

$$\Rightarrow (|z_1|^2 - 4)(|z_2|^2 - 1) = 0$$

$|z_1| = 2$ ✓ $\quad$ $|z_2| \neq 1$

$|z| \neq 0$

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if $\frac{z-1}{z+1}$ is purely imaginary, then find locus of z

$z = x+iy$

$\omega = \frac{z-1}{z+1} = \frac{(x-1)+iy}{x+iy+1} \times \frac{(x+1)-iy}{x+1-iy} = \frac{(x-1)(x+1)+y^2+i(\quad)}{(x+1)^2+y^2}$

$Re(\omega) = 0 \Rightarrow \frac{x^2-1+y^2}{(x+1)^2+y^2} = 0 \Rightarrow x^2+y^2 = 1$

$|z| = 1$

locus circle

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Complex No. in Geometry

$= |x_1 + iy_1 - x_2 - iy_2| = |x_1 - x_2 + i(y_1 - y_2)|$

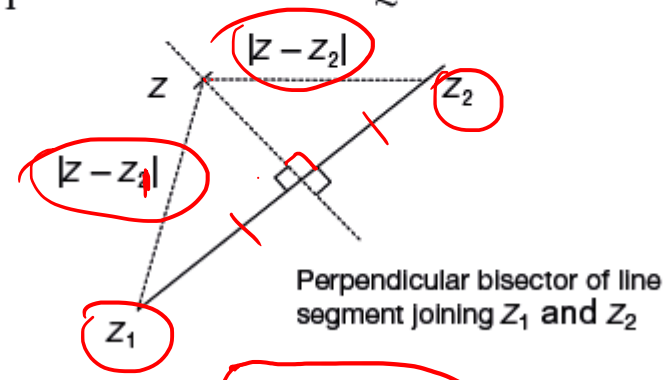
$$|z_1 - z_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$= \text{Distance between } z_1 \text{ and } z_2$$

$|z|$

$|z| = |z - 0| = \text{distance of } z \text{ from origin}$

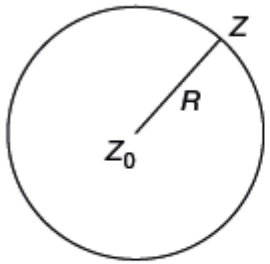
EQUATION OF THE PERPENDICULAR BISECTOR

If $P(z_1)$ and $Q(z_2)$ are two fixed points and $R(z)$ is moving point such that, it is always at equal distance from $P(z_1)$ and $Q(z_2)$, then the locus of $R(z)$ is the perpendicular bisector of PQ.



i.e., $PR = QR$ or $|z - z_1| = |z - z_2|$

Circle



Circle whose centre is Z_0 and radius R then its equation is $|Z - Z_0| = R$

$|z - z_0| < r$ represents interior of a circle $|z - z_0| = r$ and $|z - z_0| > r$ represents exterior of the circle $|z - z_o| = r$.

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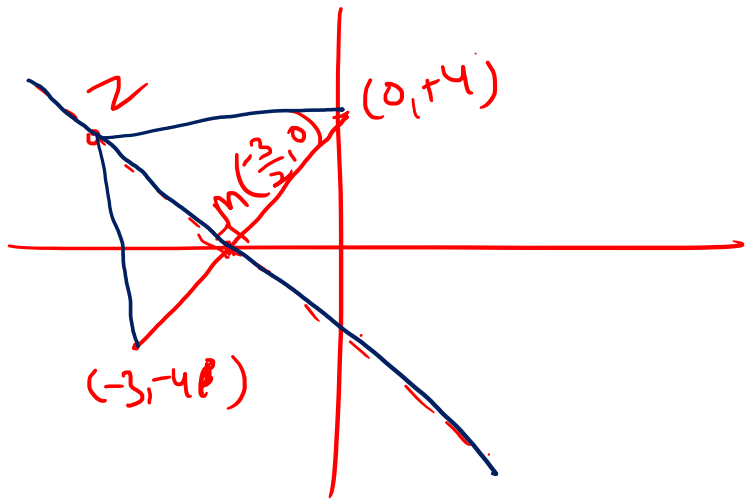
Q plot

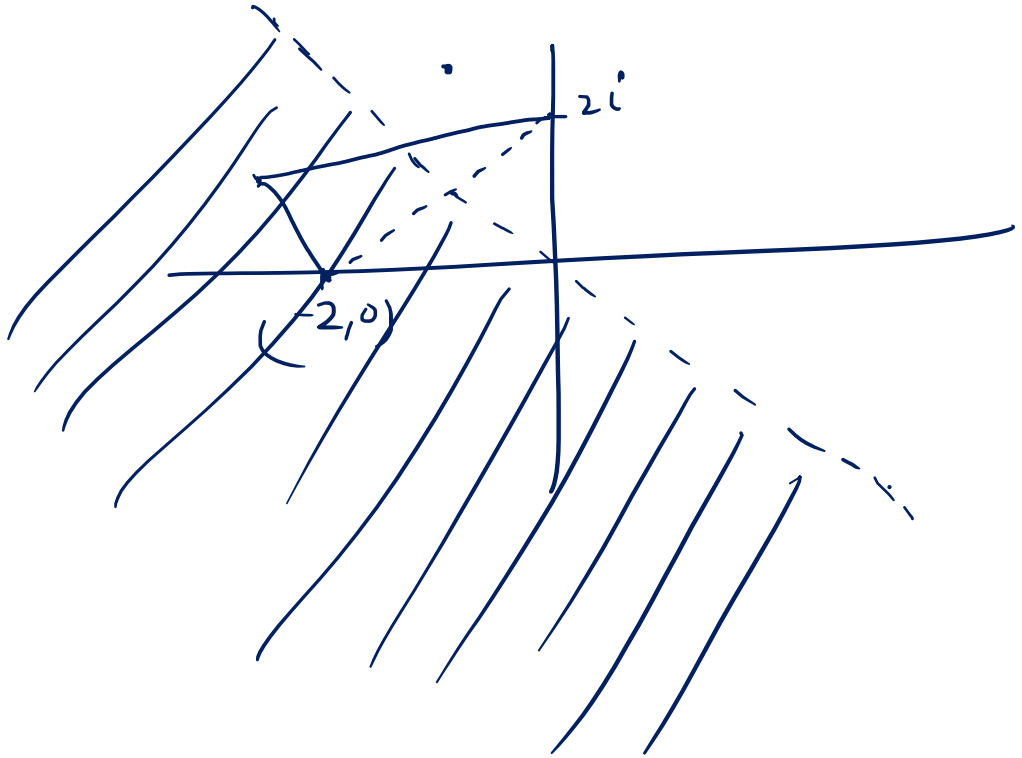
$|z - 4i| = |z + 3 + 4i|$

$|z-4i| = |z-(-3-4i)|$



$|z - 2i| > |z + 2|$

$|z-2i| > |z-(-2)|$



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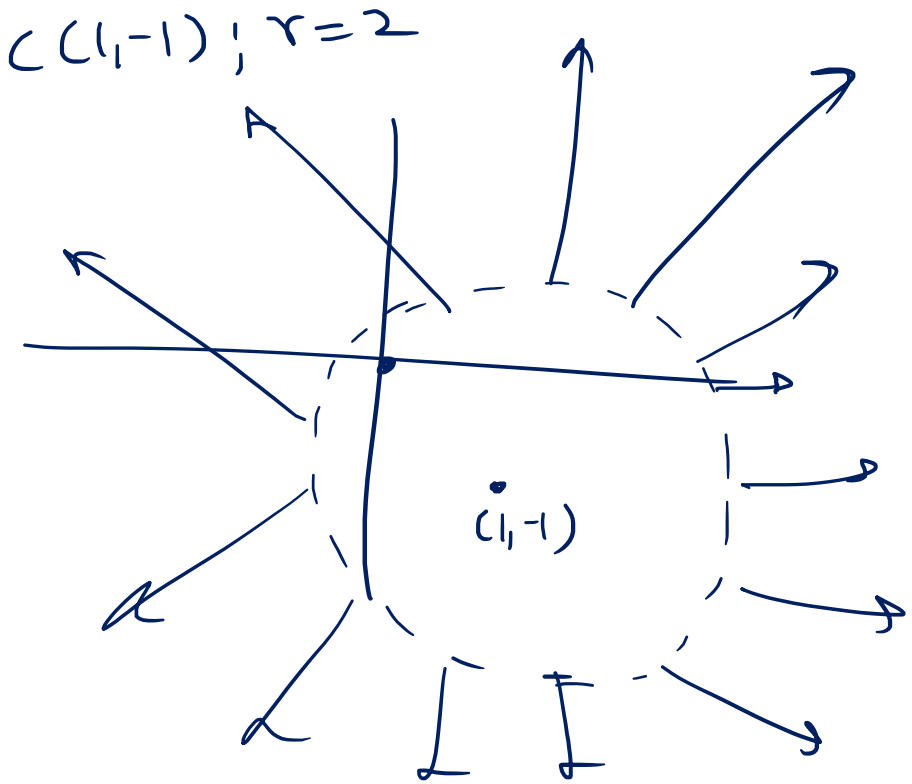
$$|z - 1 + i| > 2$$

$|z-(1-i)|>2$

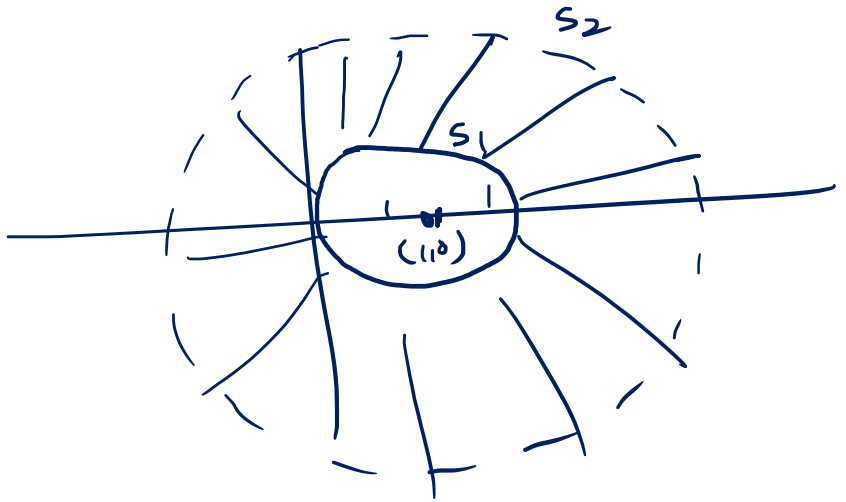


$$1 \le |z - 1| < 3$$

$C(1,0)$

S_1 $|z-1|=1$ $C_1(1,0)$ $r_1=1$

S_2 $|z-1|=3$ $C_2(1,0)$ $r_2=3$



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Complex Number

Triangular Inequality

$$\boxed{\begin{array}{l} ||z_1|-|z_2|| \le |z_1 + z_2| \le |z_1| + |z_2| \\ ||z_1|-|z_2|| \le |z_1 - z_2| \le |z_1| + |z_2| \end{array}}$$

Triangular Inequalities

$$\left||z_1| - |z_2|\right| \le |z_1 \pm z_2| \le |z_1| + |z_2|$$

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Complex Number

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Complex Number

Find minimum value of $|1 - z| + |1 + z|$

$|z_1 - z_2| \le |z_1 + z_2| \le |z_1| + |z_2|$

$z_1 = 1 - z \; ; \; z_2 = 1 + z$

$|1 - z + 1 + z| \le |1 - z| + |1 + z|$

$2 \le |1 - z| + |1 + z|$

(2)

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Complex Number

If $\left|z-\frac{2}{z}\right|=1$ find $|Z|_{max}$ & $|Z|_{min}$.

$$\left|-\frac{2}{z}\right| = \frac{|-2|}{|z|} = \frac{2}{|z|}$$

$$||z_1|+|z_2|| \le |z_1+z_2| \le |z_1| + |z_2|$$

$z_1 = z$; $z_2 = -\frac{2}{z}$

$$\left||z|-\left|\frac{2}{z}\right|\right| \le \left|z-\frac{2}{z}\right| \le |z| + \left|\frac{2}{z}\right|$$

$$\left||z|-\frac{2}{|z|}\right| \le 1 \le |z|+\frac{2}{|z|}$$

let $|z| = r > 0$

$$\left|r-\frac{2}{r}\right| \le 1 \le r+\frac{2}{r}$$

$$\left|r-\frac{2}{r}\right| \le 1$$

$$-1 \le r-\frac{2}{r} \le 1$$

(R) $1 \le r+\frac{2}{r} \Rightarrow r^2+2 \ge r$

$r^2-r+2 \ge 0$ $\quad D<0$

$\forall\, r \in R^+$

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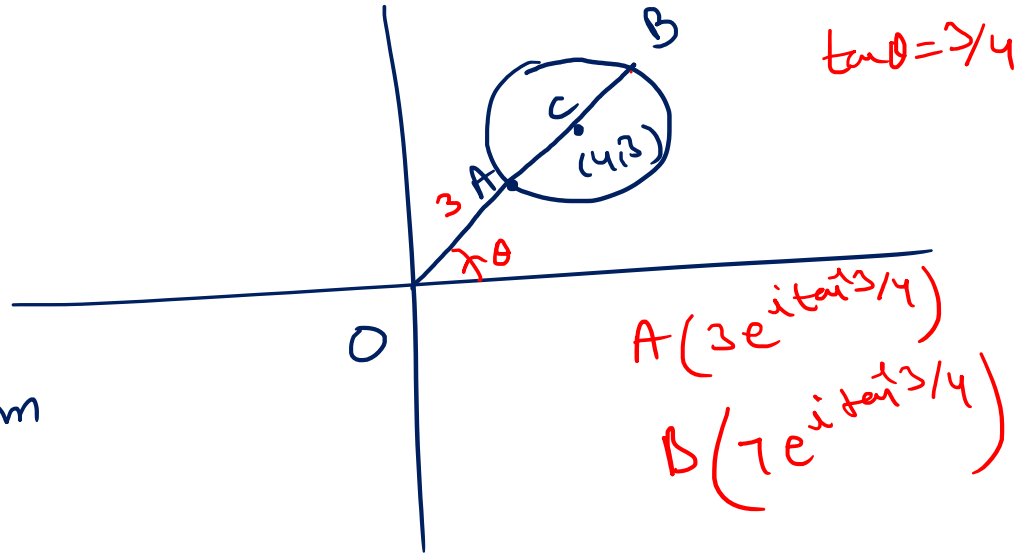
Eg

$|z - 4 - 3i| = 2$ find $|z|_{max.}$ & $|z|_{min.}$

$|z-(4+3i)| = 2$

C (4, 3)



$|z|$ = distance from origin

OC = 5 r = 2

OA = 3 $= |z|_{min}$

OB = 7 $= |z|_{max}$

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Complex Number

If $|z_1| = 6$, $z_2 = 24 + 7i$ then find the maximum and minimum value of $|z_1 + z_2|$ and also find the corresponding of complex number z_1.

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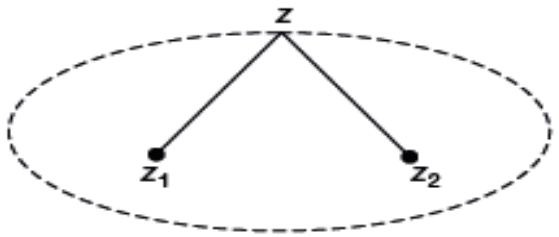
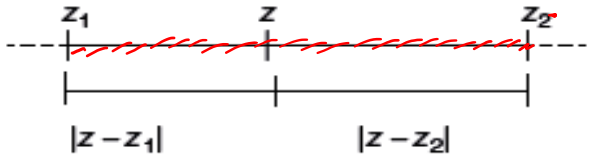
$|z-z_1|+|z-z_2|=K$

- $K>|z_1-z_2|$
  - Locus of $z \rightarrow$ Ellipse (z_1 and z_2 are foci)



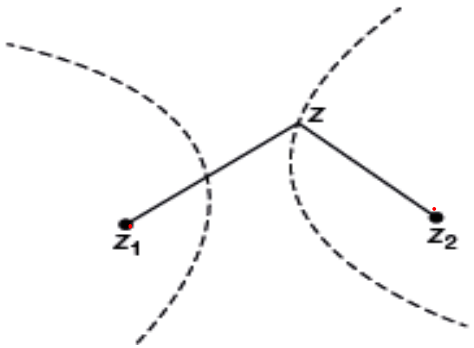
- $K=|z_1-z_2|$
  - z lies on line segment joining z_1 and z_2



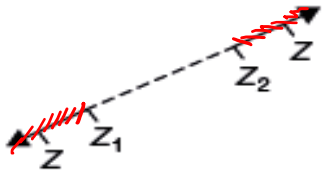
- $K<|z_1-z_2|$
  - $z \in \phi$

$||z-z_1|-|z-z_2||=K$

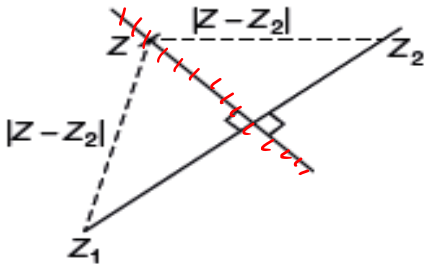
- $K<|z_1-z_2|$
  - Locus of $z \rightarrow$ Hyperbola with foci z_1 and z_2



- $K=|z_1-z_2|$
  - $z \rightarrow$ A pair of rays



- $K=0$
  - $|z-z_1|=|z-z_2|$
  - $z \rightarrow$ perpendicular bisector of line joining z_1 & z_2



- $K>|z_1-z_2|$
  - $z \in \varphi$

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Identify locus of z if

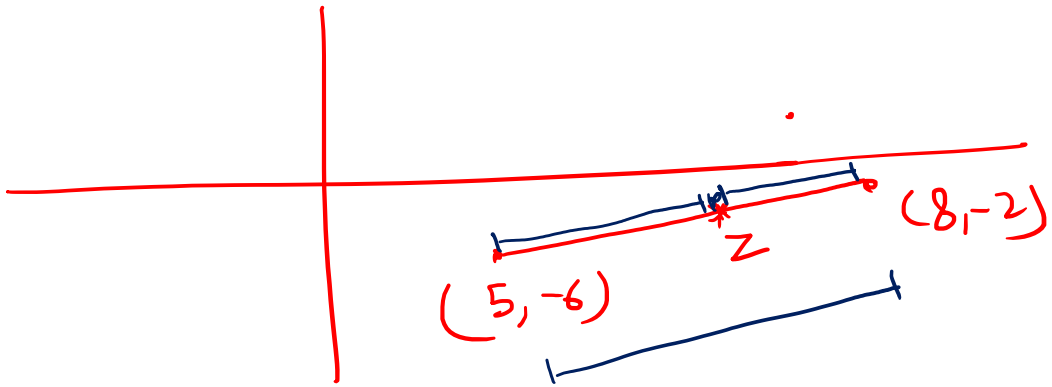
$$|z - 8 + 2i| + |z - 5 + 6i| = 5$$

Hyp Rays

$|z-(8-2i)| + |z-(5-6i)| = 5$

$A(8,-2)$ $B(5,-6)$ $AB = \sqrt{9+16} = 5$

Δs



z → line segment joining (5,-6) & (8,-2)

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Identify locus of z if

$|z - 4i| + |z + 4i| = 10$

$z \rightarrow$ ellipse

$A(0,4)$ $B(0,-4)$

$AB = 8 =$ third side

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Identify locus of z if $\|z-3+4i|-|z+2-i\|=1$

$||z-(3-4i)|-|z-(-2+i)||=1$

$A(3,-4)$ & $B(-2,1)$

$AB=\sqrt{25+25}=5\sqrt{2}$ = third

$\Rightarrow$ Hyperbola.

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Solve $z^2 + |z| = 0$

$z = x + iy$

$$(x+iy)^2 + \sqrt{x^2+y^2} = 0$$

$$x^2 - y^2 + \sqrt{x^2+y^2} + 2xyi = 0 + i0$$

$$x^2 - y^2 + \sqrt{x^2+y^2} = 0 \quad \& \quad xy = 0$$

$x = 0$	$y = 0$
$0 - y^2 + \sqrt{y^2} = 0$	$x^2 + \sqrt{x^2} = 0$

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Find locus of z if $\mathrm{Re}\left(\frac{1}{z}\right) < \frac{1}{2}$

$$\frac{1}{z} = \frac{1}{x+iy} \times \frac{x-iy}{x-iy} = \frac{x-iy}{x^2+y^2}$$

$$\frac{x}{x^2+y^2} < \frac{1}{2} \Rightarrow 2x < x^2+y^2$$

$$\Rightarrow x^2+y^2-2x > 0$$

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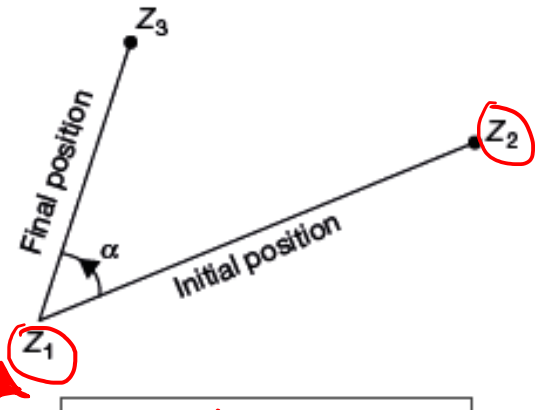
Rotation Theorem (Coni Method)

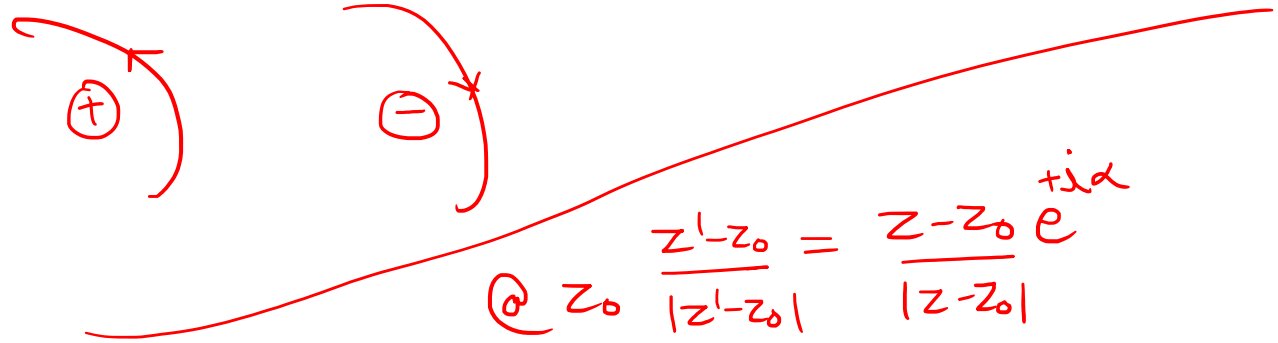
Rotation About Z_1



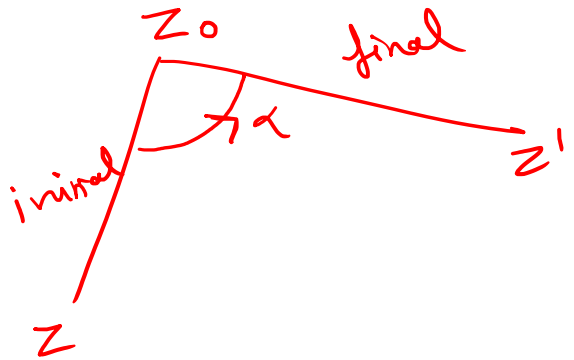
$$\left(\frac{z_3 - z_1}{|z_3 - z_1|}\right) = \left(\frac{z_2 - z_1}{|z_2 - z_1|}\right) e^{i\alpha}$$

Final position | Initial position

α → with sign



@ z_0 $\quad \frac{z' - z_0}{|z' - z_0|} = \frac{z - z_0}{|z - z_0|} e^{+i\alpha}$



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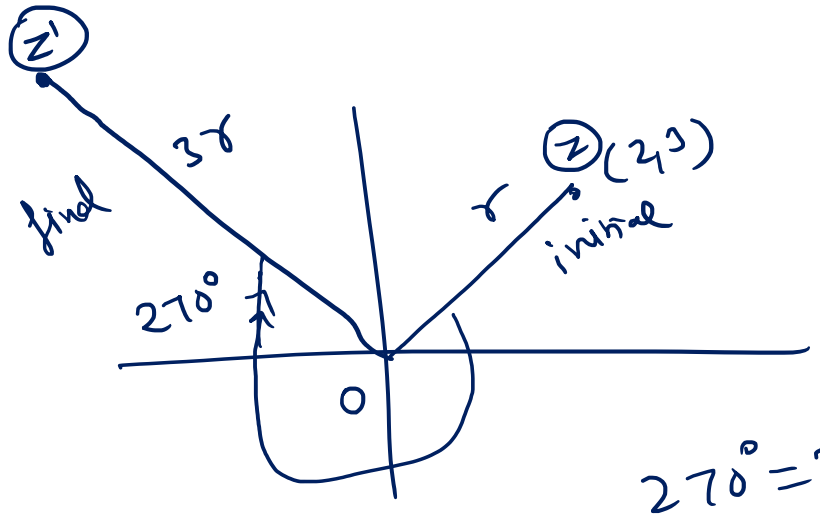
The vector z = 2 + 3i is turned through and angle 270° in clockwise direction and stretched 3 times. Find the vector in new position



$270^\circ = \frac{3\pi}{2}$

$|z| = r$; $|z'| = 3|z|$

Q 0

$$\frac{z'-0}{|z'-0|} = \frac{z-0}{|z-0|} e^{i(-3\pi/2)}$$

$$\frac{z'}{3|z|} = \frac{2+3i}{|z|} \left(0 + i\sin\left(\frac{-3\pi}{2}\right)\right)$$

$$z' = (6+9i)(i)$$

$$z' = -9+6i$$

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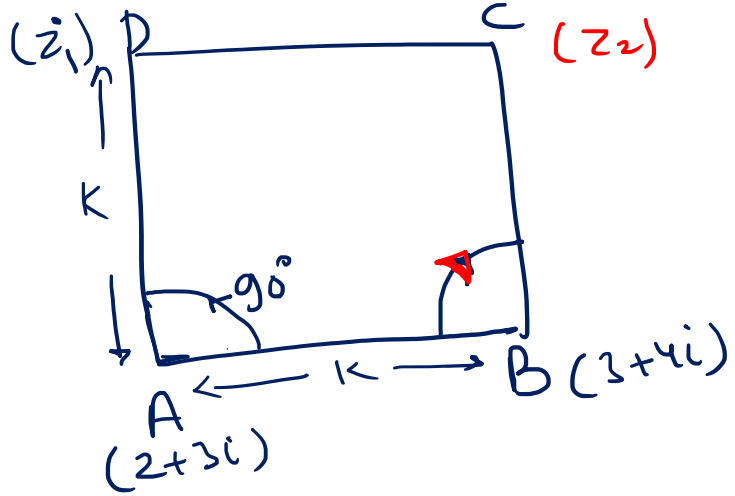
Eg

If A(2 + 3i) and B(3 + 4i) are two vertices of a square ABCD (take in anticlockwise order) then find C and D.



Q A

$$\frac{z_1-2-3i}{AD} = \frac{3+4i-2-3i}{AB}\, e^{+i\pi/2}$$

$$z = 2+3i+(1+i)i$$

$$= 2+3i+i-1$$

$$z = 1+4i$$

D(1+4i)

Q B

$$\frac{z_2-3-4i}{BC} = \frac{2+3i-3-4i}{AB}\, e^{-i\pi/2}$$

$$z_2 = 3+4i+(-1-i)(-i)$$

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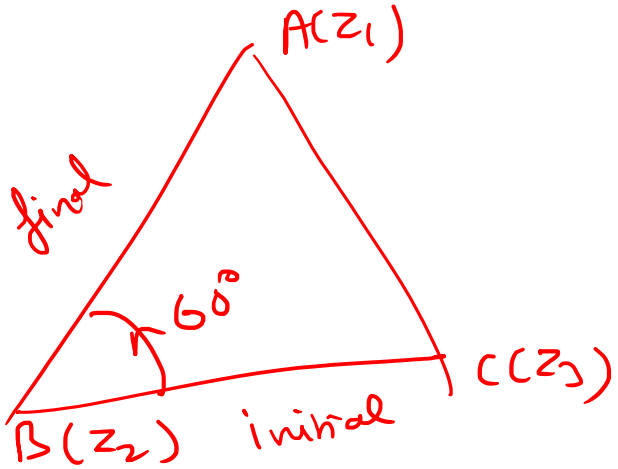
If z_1, z_2, z_3 be the vertices of a triangle, then the triangle is equilateral iff

$$z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$$



@B

$$\frac{z_1 - z_2}{BA} = \frac{z_3 - z_2}{BC} e^{i\pi/3} \quad \text{—(1)}$$

—(2)

@C

(1)/(2)

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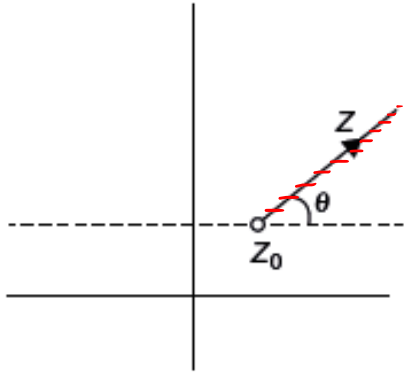
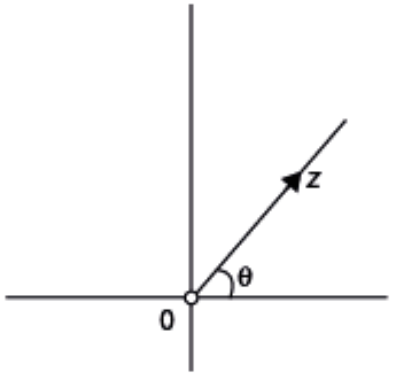
Locus based on arguments

LOCUS PROBLEMS ON ARGUMENT

* $\text{Arg}(z - z_0) = \theta$ = A ray originated from z_0 at an angle θ with positive real axis, where z_0 is excluded.



$\text{Arg}(z) = \text{Arg}(z - 0) = \theta$



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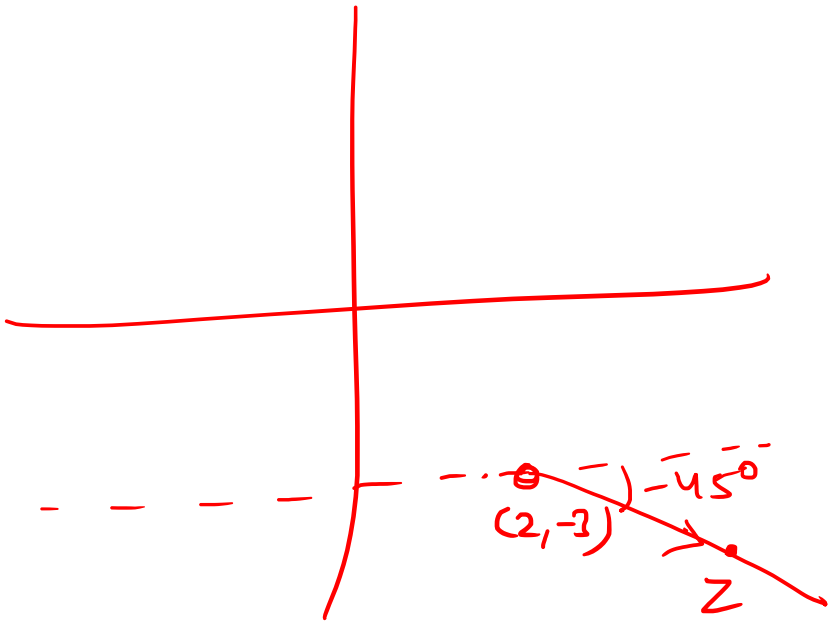
Complex Number

Plot locus of z on argand plane

if $\arg(z - 2 + 3i) = -\frac{\pi}{4}$

$\arg(z - (2-3i)) = -45^\circ$

$(2-3i) = (2, -3)$



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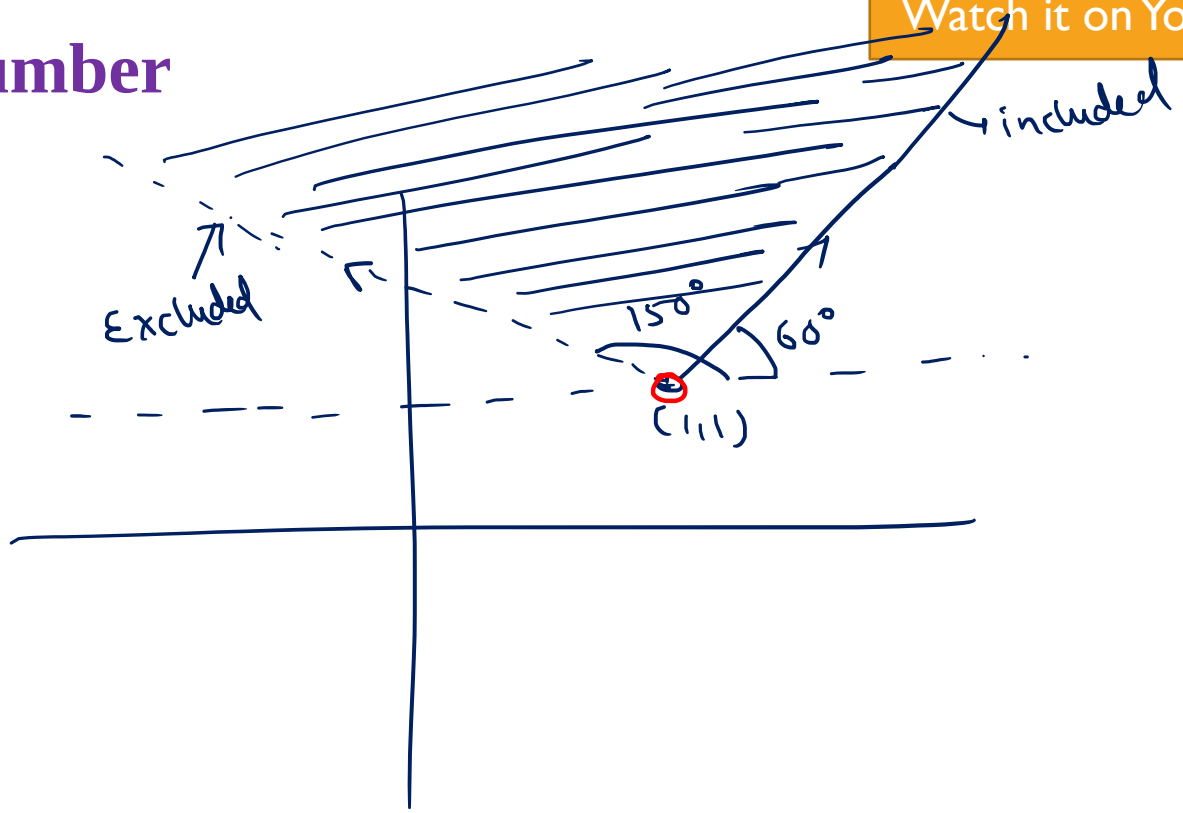
Complex Number

Draw the region satisfied by z on argand plane

if $\frac{\pi}{3} \le \arg(z - 1 - i) < \frac{5\pi}{6}$

$\frac{\pi}{3} \le \arg(z-(1+i)) < \frac{5\pi}{6}$



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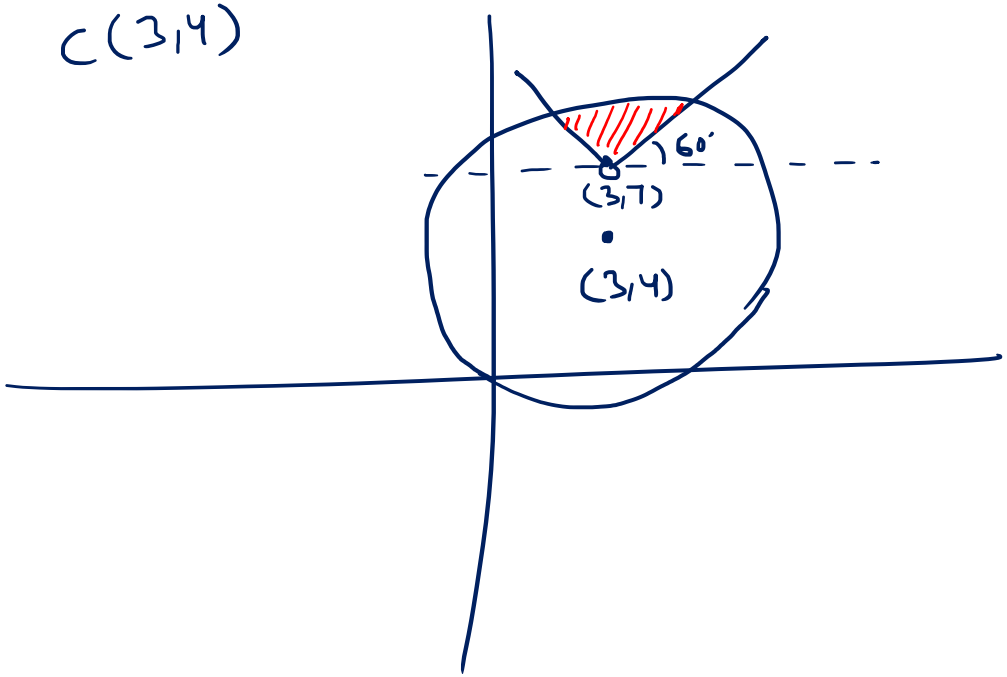
Draw region

$$|z - 3 - 4i| \le 5 \;\&\; \frac{\pi}{3} \le \arg(z - 3 - 7i) \le \frac{2\pi}{3}$$

$|z-(3+4i)| \le 5$ $\qquad$ $z-(3+7i)$

$C(3,4)$



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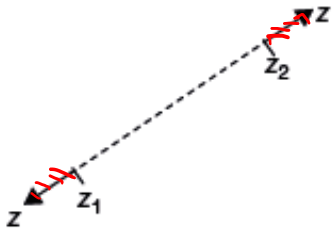
ARGUMENT BASED LOCUS PROBLEMS

Locus of z, if $\text{Arg}\left(\frac{z-z_1}{z-z_2}\right)=\theta$

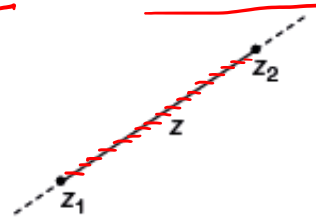
$\text{Arg}\left(\frac{z-z_1}{z-z_2}\right)=\theta$

$\Rightarrow \arg\left(\frac{z_1-z}{z_2-z}\right)=\underset{\text{final}}{\text{Arg}(z_1-z)}-\underset{\text{initial}}{\text{Arg}(z_2-z)}=\theta$

Case (I): $\theta=0\,;z\to$ pair of rays



Case (II): $\theta=\pi\,;z\to$ line segment joining z_1 and z_2.



Case (III): $0<\text{Arg}\left(\frac{z-z_1}{z-z_2}\right)<\pi\,;z\to$ part of a circle.

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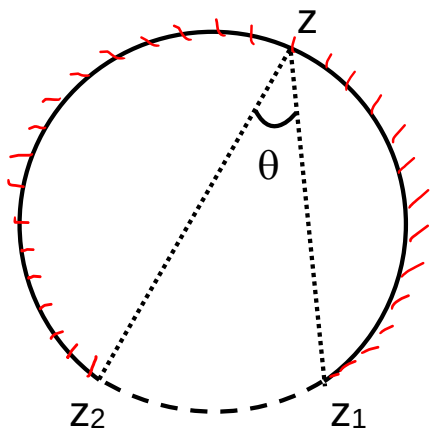
Complex Number

$$\theta = \arg\left(\frac{z - z_1}{z - z_2}\right)$$

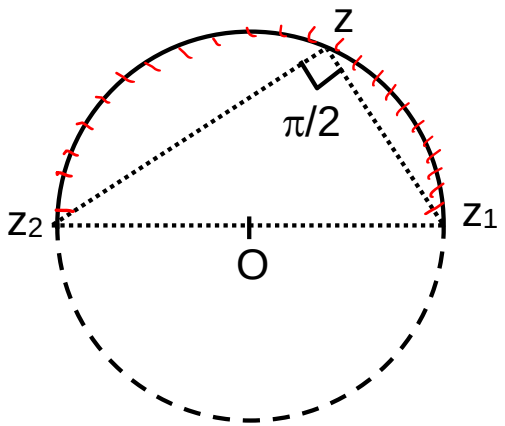
$0 < \theta < \frac{\pi}{2}$

$z \rightarrow$ major arc of a circle



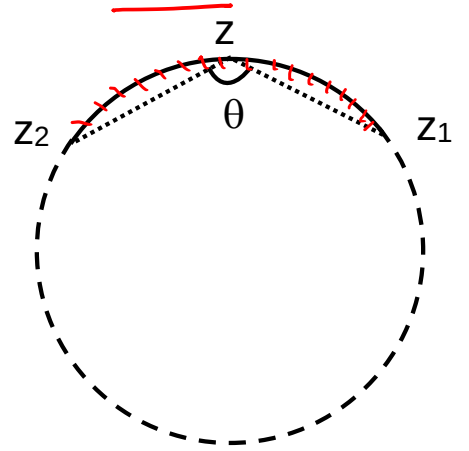
$\theta = \frac{\pi}{2}$

$z \rightarrow$ semi circle



$\frac{\pi}{2} < \theta < \pi$

$z \rightarrow$ minor arc of a circle



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Complex Number

$$\arg\left(\frac{z-3+5i}{z-1-i}\right)=\pi$$

$\arg\left(\frac{z-(3-5i)}{z-(1+i)}\right)=\pi$

z - line segment (1,1) & (3,-5)

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Complex Number

$$\arg\left(\frac{z-3+4i}{z-2-5i}\right)=\frac{5\pi}{6}$$

$z \rightarrow$ minor arc of circle

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Find locus of the points z which satisfy the condition

$$\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{3}$$

z → major arc

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Let $z_1 = 10 + 6i$ and $z_2 = 4 + 6i$. If z is any complex number such that the argument of $\frac{(z - z_1)}{(z - z_2)}$ is $\frac{\pi}{4}$,

then prove that $|z - 7 - 9i| = 3\sqrt{2}$ **[IIT-1991]**

major arc of circle

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Straight Line

put $z = x + iy$

The general equation of a straight line is of $\bar{\alpha}z + \alpha\bar{z} + k = 0$.

where α is complex number and k is real number.

Real slope of the line $m = -\dfrac{\mathrm{Re}(\alpha)}{\mathrm{Im}(\alpha)} = -i\dfrac{(\alpha+\bar{\alpha})}{(\alpha-\bar{\alpha})}$.

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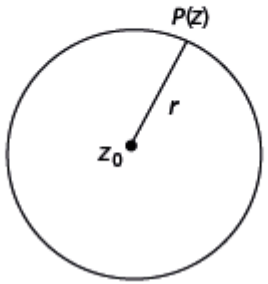
Circle

(i) The equation of a circle whose centre is at point having affix z_o and radius r is $|z-z_0|=r$ centre (z_0) and radius $=r$.



If the centre of the circle is at origin and radius r, then its equation is $|z|=r$.

(ii) $|z-z_0|<r$ represents interior of a circle $|z-z_0|=r$ and $|z-z_0|>r$ represents exterior of the circle $|z-z_o|=r$.

(iii) General equation of a circle: The general equation of the circle is $z\bar{z}+\alpha\bar{z}+\bar{\alpha}z+k=0$ with centre $(-\alpha)$ and radius $=\sqrt{\alpha\bar{\alpha}-k}$

Therefore, centre and radius are $-a$ and $\sqrt{|\alpha|^2-k}$ respectively.

Working method (substitute $z=x+iy$)

(iv) Equation of circle in diametric form:

If the endpoints of the diameter represented by $A(z_1)$ and $B(z_2)$ and $P(z)$ be any point on the circle then, $|z-z_1|^2+|z-z_2|^2=|z_1-z_2|^2$ on simplification $(z-z_1)(\bar{z}-\bar{z}_2)+(z-z_2)(\bar{z}-\bar{z}_1)=0$ which is required equation of circle in diametric form.

(v) Argument form of circle: Arg $\left(\frac{z-z_1}{z-z_2}\right)=\theta$; $\theta\neq 0,\pi$ then z is a part of a circle

$Arg\left(\frac{z-z_1}{z-z_2}\right)=\pm\pi/2$ then $z\to$ z is a semicircle with z_1 and z_2 as the extremities of diameter.

(vi) Apollonius circle form: $\left|\frac{z-z_1}{z-z_2}\right|=k$ where $k\neq 1; k>0$, then locus of z is circle Working method (substitute $z=x+iy$)

$z\bar{z}+\alpha\bar{z}+\bar{\alpha}z+k=0$

centre $(-\alpha)$, $r=\sqrt{\alpha\bar{\alpha}-k}$

put $z=x+iy$

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$z\bar{z} + (1-i)z + (1+i)\bar{z} - 7 = 0$ find centre & radius

put $z = x+iy$

$x^2+y^2 + (1-i)(x+iy) + (1+i)(x-iy) - 7 = 0$

$x^2+y^2 + x + y + x + y - 7 = 0$

$x^2+y^2+2x+2y-7=0$

$C(-1,-1)$ $r = \sqrt{1+1+7} = 3$

$-\alpha$ $r = \sqrt{\alpha\bar{\alpha} - k}$

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$z\bar{z} + (-3 + 4i)\bar{z} - (3 + 4i)z + k = 0$.

Find k for which it represents circle.

put $z = x + iy$

$r = \sqrt{\alpha\bar{\alpha} - k}$

$\alpha\bar{\alpha} - k > 0$

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$\left|\frac{Z+4i}{Z-i}\right| = 2$, find centre & radius of circle.

put $z = x+iy$

$\left|\frac{x+iy+4i}{x+iy-i}\right| = 2 \Rightarrow \sqrt{x^2+(y+4)^2} = 2\sqrt{x^2+(y-1)^2}$

square it

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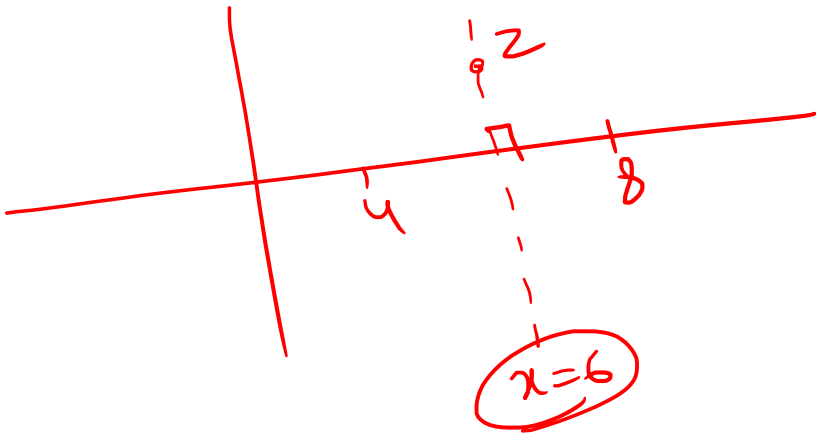
Find 'z' if $\left|\frac{z-4}{z-8}\right| = 1$ & $\left|\frac{z-12}{z-8i}\right| = \frac{5}{3}$

$|z-4| = |z-8|$

put $z = x+iy$

$x = 6$

$z = 6+iy$



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Solve $2z = |z| + 2i$

put $z = x + iy$

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Roots of Unity

Cube Root of Unity

$$z^3 - 1 = 0 \Rightarrow (z-1)(z^2+z+1) = 0, z \neq 1$$

$$z^2 + z + 1 = 0 \Rightarrow z = -\left(\frac{1 \pm \sqrt{3}i}{2}\right)$$

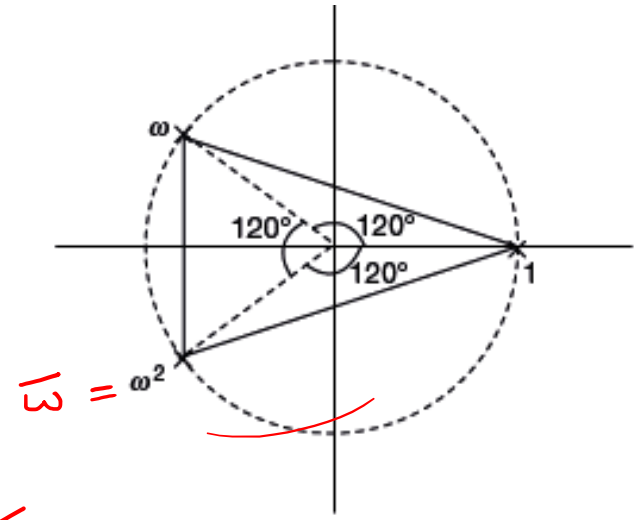
$$z = 1; z = \frac{-1+\sqrt{3}i}{2} = e^{i2\pi/3} = \omega$$

$$z = \frac{-1-\sqrt{3}i}{2} = e^{-i2\pi/3} = e^{i4\pi/3} = \bar{\omega} = \omega^2$$

$1 + \omega + \omega^2 = 0$

$\omega^3 = 1$

$\omega \rightarrow$ imaginary cube root of unity



$z^2 + z + 1 = 0$
$t^2 + t + 1 = 0$
roots ω, ω^2

- Cube roots of unity are in GP with common ratio ω.
- All three roots lie on a circle and equally divides the circle. Hence, they construct an equilateral triangle.
- $1 + \omega^n + \omega^{2n} = \begin{cases} 0, \text{if } n \text{ is not a multiple of } 3 \\ 3, \text{if } n \text{ is a multiple of } 3 \end{cases}$
- A complex number $a + ib$, for which $|a:b| = 1:\sqrt{3}$ or $\sqrt{3}:1$, can always be expressed in terms of $1, \omega, \omega^2$.
- Cube root of -1 are $-1, -\omega, -\omega^2$.

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nth root of unity

$z^n = 1 \Rightarrow z = (1)^{1/n} = (e^{i0})^{1/n}$ (change Euler form)

$z = (e^{i2m\pi})^{1/n}$ (add $2m\pi$ in argument)

$z = e^{\frac{i2m\pi}{n}}$ where, $m = 0, 1, 2, \ldots\ldots n-1$

$m = 0$ $\quad z = 1$

$m = 1$ $\quad z = e^{i2\pi/n} = \alpha_1 = \alpha$

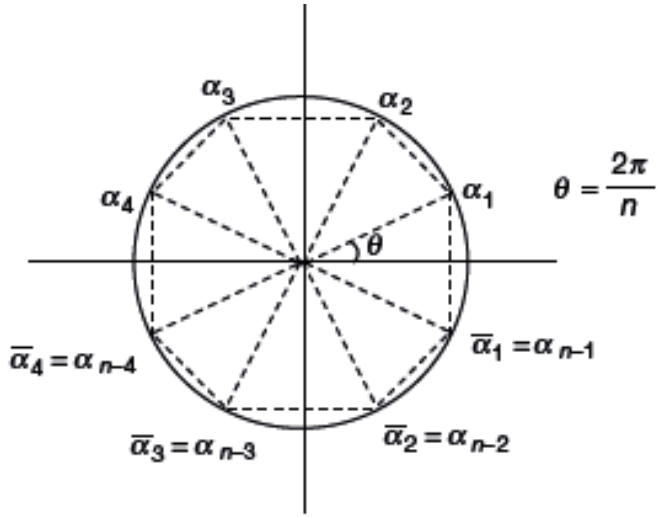
$m = 2$ $\quad Z = e^{i4\pi/n} = \alpha_2 = \alpha^2$

$m = 3$ $\quad z = e^{i6\pi/n} = \alpha_3 = \alpha^3$

$m = n-1$ $\quad z = e^{i\frac{2\pi(n-1)}{n}} = \alpha_{n-1} = \alpha^{n-1}$

GP with common ratio $(\alpha = e^{i2\pi/n})$

- n^{th} root of unity are in GP with common ratio (α)
- All n roots lie on unit radii circle with centre at origin.
- All n roots are equidistant on circle. That means they divide circle into n equal parts.
- All n roots construct regular n sided polygon
- All imaginary roots are present with their conjugates.



Equation $z^n - 1 = 0$ has roots $1, \alpha_1, \alpha_2, \alpha_3 \ldots \alpha_{n-1}$

Sum of roots $= 1 + \alpha_1 + \alpha_2 + \alpha_3 + \ldots\ldots + \ldots\alpha_{n-1} = 0$

Product of roots $= 1.\alpha_1.\alpha_2.\alpha_3.\ldots\ldots\ldots.\alpha_{n-1} = (-1)^{n-1}$

$z^3 - 1 = 0$ has roots $1, \omega, \omega^2$	$z^4 - 1 = 0$ has roots $1, -1, i$ and -1
Product of roots $= \omega^3 = 1$	Product of roots $= 1(-1)$ $i(-i) = -1$

$1^P + \alpha_1^P + \alpha_2^P + \alpha_3^P + \ldots\alpha_{n-1}^P =$

$\begin{cases} n, \text{ if } P \text{ is multiple of } n \\ 0, \text{ if } P \text{ is not multipled } n \end{cases}$

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If ω is imaginary cube root of unity, then find value of

(a) $(1 + \omega^2 - \omega)(1 + \omega - \omega^2)$

$= (-\omega - \omega)(-\omega^2 - \omega^2)$

$= (-2\omega)(-2\omega^2)$

$= 4\omega^3 = 4$

(b) $(1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5$

$= (1 + \omega^2 - \omega)^5 + (-\omega^2 - \omega^2)^5$

$= (-\omega - \omega)^5 + (-2\omega^2)^5$

$= -32\omega^5 - 32\omega^{10}$

$= -32(\omega^5 + \omega^{10})$

$= -32(\omega^2 + \omega)$

$= -32(-1) = 32$

$\omega = \frac{-1}{2} + \frac{\sqrt{3}i}{2}$ $r = 1$

$\alpha = \tan^{-1}\sqrt{3} = \pi/3$

$(-1/2, \sqrt{3}/2)$ IInd

$\theta = \pi - \alpha = 2\pi/3$

$\omega = e^{i2\pi/3}$

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If ω is complex cube root of unity then evaluate

$$\frac{1}{1+2\omega}+\frac{1}{2+\omega}-\frac{1}{1+\omega}$$

$1+\omega+\omega^2=0$

$\omega^3=1$

$$=\frac{1}{1+2\omega}+\frac{1+\omega-2-\omega}{(2+\omega)(1+\omega)} = \frac{1}{1+2\omega}+\frac{-1}{2+\omega+2\omega+\omega^2}$$

$$= \frac{1}{1+2\omega}-\frac{1}{2+3\omega+(-1-\omega)}$$

$$= \frac{1}{1+2\omega}-\frac{1}{1+2\omega}=0$$

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Evaluate $\sum_{r=0}^{10}(1+\omega^r+\omega^{2r})$ where ω is complex cube root of unity.

$$= \sum_{r=0}^{10} 1 + \sum_{r=0}^{10}\omega^r + \sum_{r=0}^{10}\omega^{2r}$$

$\omega^{3n}=1$

$$= 11 + 1\left(\frac{\omega^{11}-1}{\omega-1}\right) + 1\cdot\left(\frac{(\omega^2)^{11}-1}{\omega^2-1}\right)$$

$\omega^{22}=\omega^{21}\cdot\omega=\omega$

$$= 11 + \left(\frac{\omega^2-1}{\omega-1}\right) + \left(\frac{\omega-1}{\omega^2-1}\right)$$

$$= 11 + \omega+1 + \frac{1}{\omega+1} = 12+\omega+\left(\frac{1}{-\omega^2}\right)\times\frac{\omega}{\omega}$$

$$= 12+\omega+\frac{\omega}{(-\omega^3)} = 12+\omega-\omega = 12$$

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If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009}$ is equal to **[AIEEE-2010]**

(A) –2 (B) –1 (C) 1 (D) 2

$t^2 + t + 1 = 0 \rightarrow \omega, \omega^2$

$-\omega, -\omega^2$

$(-\omega)^{2009} + (-\omega^2)^{2009}$

$= -[\omega^{2009} + \omega^{4018}]$

$= -[\omega^2 + \omega] = 1$

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If $\omega(\neq 1)$ is a cube root of unity and $(1 + \omega)^7 = A + B\omega$. Then (A, B) is equal to **[AIEEE-2011]**

(A) (1, 1) (B) (1, 0) (C) (–1, 1) (D) (0, 1)

$(-\omega^2)^7 = -\omega^{14} = -\omega^2 = 1+\omega = A+B\omega$

$A = B = 1$

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Evaluate

$(2-\omega)(2-\omega^2)+2(3-\omega)(3-\omega^2)+ \ldots\ldots\ldots + (n-1)(n-\omega)(n-\omega^2)$ where ω is complex cube root of unity & $n \in N$ **[IIT 1996]**

$$\sum_{n=2}^{n} (n-1)(n-\omega)(n-\omega^2) \Longrightarrow n^3-1$$

$$= \sum_{n=2}^{n} n^3-1 = \left[\left(\frac{n(n+1)}{2}\right)^2-1\right]-(n-1)$$

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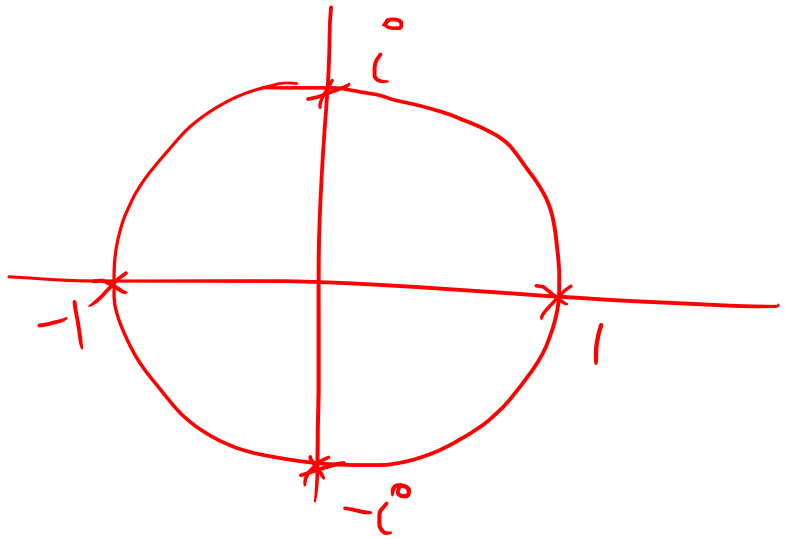
4th root of unity

$z^4 = 1$

$z = (1)^{\frac{1}{4}}$ → 1, i, -1, $-i$



$i^4 = 1$

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n^{th} root of unity

$z^n = 1$

$z = (1)^{1/n}$

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If ω is imaginary 5^{th} root of unity, find $\omega^4 + \omega^8 + \omega^{12} + \omega^{16}$

$\omega^5 = 1$

$1 + \omega + \omega^2 + \omega^3 + \omega^4 = 0$

$\omega^4 + \omega^3 + \omega^2 + \omega^1$

$= -1$

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If ω is real 5^{th} root of unity find $\omega^4 + \omega^8 + \omega^{12} + \omega^{16}$

$\omega = 1$

$1 \quad 1 \quad 1 \quad 1$

$= 4$

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Find value of $\sum_{\lambda=1}^{12}\left(\sin\frac{2\lambda\pi}{13}-i\cos\frac{2\lambda\pi}{13}\right)$

$=$

$= (-i)\sum_{\lambda=1}^{12}\left(\frac{1}{-i}\sin\frac{2\lambda\pi}{13}+\cos\frac{2\lambda\pi}{13}\right)$

$-\frac{1}{i}=i$

$\alpha = e^{i2\pi/n}$

$= -i\left(\sum_{\lambda=1}^{12} e^{i\frac{2\lambda\pi}{13}}\right)$

$= -i(\alpha_1+\alpha_2+\cdots+\alpha_{12})$

$-1 = i^2$

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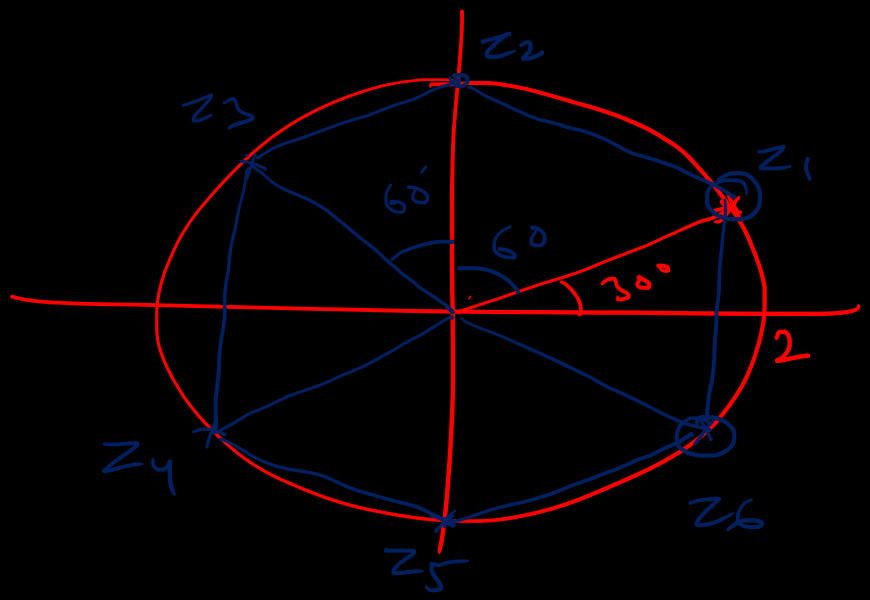
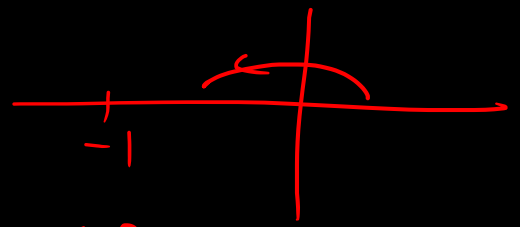
$$z^6 = -64 = -2^6 \Rightarrow z = 2(-1)^{1/6} = 2(e^{i\pi})^{1/6} = 2e^{i\pi/6}$$

(2)





$$\theta = \frac{360^\circ}{6} = 60^\circ$$

regular hexagon.

$z = x + iy$

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If 1, α_1, α_2, α_3,……, α_{n-1} are n roots of n^{th} root of unify then evaluate the following results.

(a) $(5-\alpha_1)(5-\alpha_2)\ldots\ldots(5-\alpha_{n-1})$

(b) $\frac{1}{5-\alpha_1}+\frac{1}{5-\alpha_2}+\ldots+\frac{1}{5-\alpha_{n-1}}$

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